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Institute of Mathematics of the National Academy of Sciences of Ukraine
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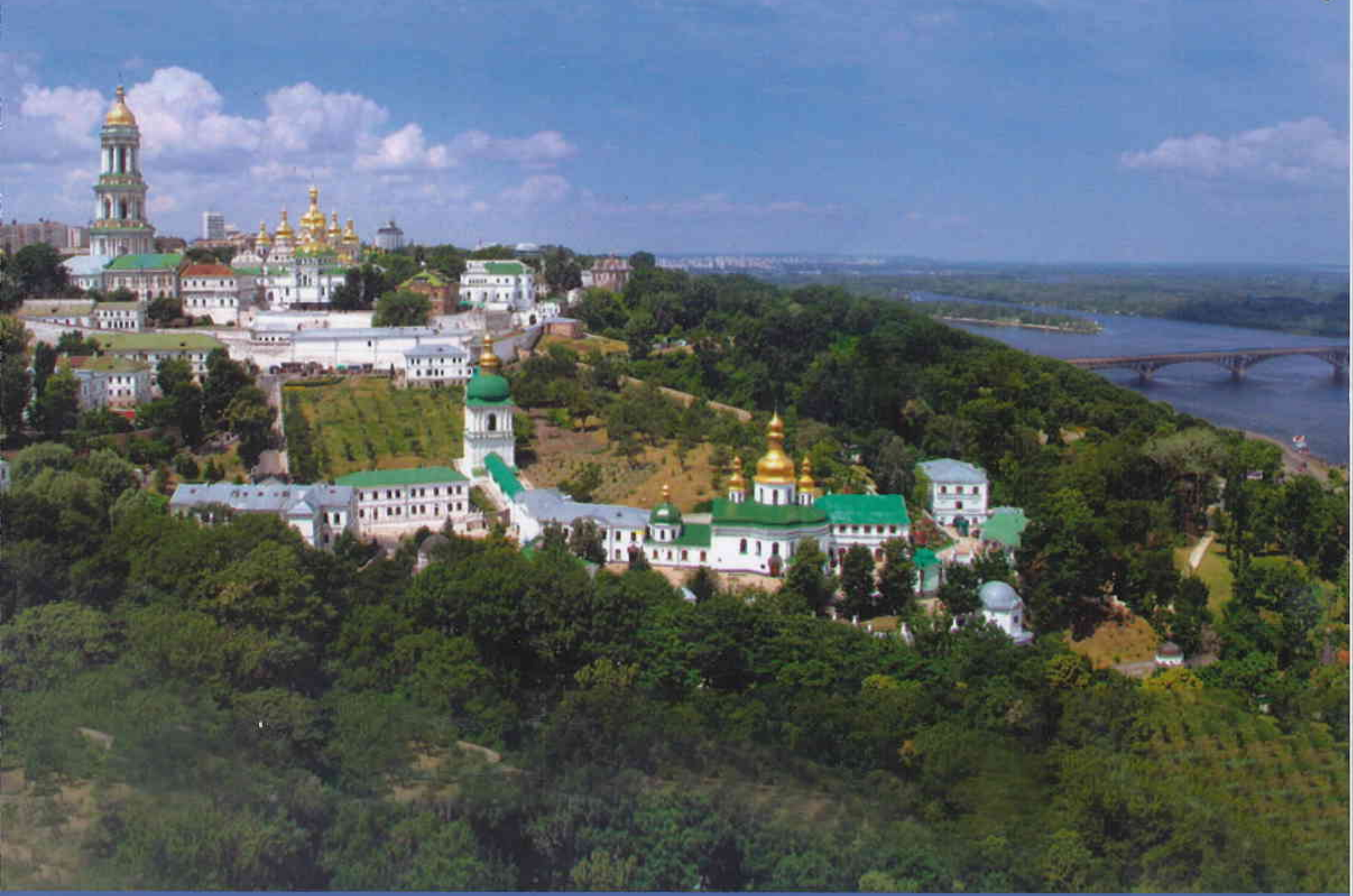

Book of Abstracts

5th INTERNATIONAL CONFERENCE

of Young Scientists
on Differential Equations and Applications
dedicated to Yaroslav Lopatynsky

9-11 November, 2016, Kyiv, Ukraine

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Existence and uniqueness of entropy solution for nonlinear elliptic degenerate anisotropic equations

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Let Ω be a bounded domain in $\mathbb{R}^n$ with a boundary $\partial\Omega$, $n \geqslant 2$. Assume that $1 < q_i < n$ are real numbers, and ν_i are nonnegative functions in Ω such that $\nu_i > 0$ a.e. in Ω, $\nu_i \in L^1_{\rm loc}(\Omega)$, $\nu_i^{-1/(q_i-1)} \in L^1(\Omega)$, $i = 1, \dots, n$. Suppose Carathéodory functions $a_i : \Omega \times \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, n$, satisfy the conditions of growth, strict monotonicity and following coercitivity condition: $\sum_{i=1}^n a_i(x,\xi)\xi_i \geqslant c\sum_{i=1}^n \nu_i|\xi_i|^{q_i} - g(x)$; here c is a positive constant, and $g \in L^1(\Omega)$ is a nonnegative function. Put $q = \{q_1, \dots, q_n\}$, $\nu = \{\nu_1, \dots, \nu_n\}$. We define $W^{1,q}(\nu,\Omega) = \{u \in L^1(\Omega) : \nu_i|D_i u|^{q_i} \in L^1(\Omega), i = 1, \dots, n\}$. $W^{1,q}(\nu,\Omega)$ is a Banach space with respect to the norm $\|u\|_{W^{1,q}(\nu,\Omega)} = \|u\|_{L^1(\Omega)} + \sum_{i=1}^n \left(\int_\Omega \nu_i|D_i u|^{q_i}\,dx \right)^{1/q_i}$.

Denote by $\overset{\circ}{W}{}^{1,q}(\nu,\Omega)$ the closure of $C_0^\infty(\Omega)$ in $W^{1,q}(\nu,\Omega)$. Let $F : \Omega \times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function. We consider the Dirichlet problem:

$$-\sum_{i=1}^n \frac{\partial}{\partial x_i} a_i(x, \nabla u) = F(x,u) \quad \text{in } \Omega, \tag{1}$$

$$u = 0 \quad \text{on } \partial\Omega. \tag{2}$$

Definition. *An entropy solution of problem* (1), (2) is a function $u : \Omega \to \mathbb{R}$ such that: 1) $T_k(u) \in \overset{\circ}{W}{}^{1,q}(\nu,\Omega)$, where T_k is a standard cut function of the level $k > 0$;
2) $F(x,u) \in L^1(\Omega)$;
3) if $w \in \overset{\circ}{W}{}^{1,q}(\nu,\Omega) \cap L^\infty(\Omega)$, $k \geqslant 1$, and $l \geqslant k + \|w\|_{L^\infty(\Omega)}$, then

$$\int_\Omega \left\{ \sum_{i=1}^n a_i(x, \nabla T_l(u)) D_i T_k(u-w) \right\} dx \leqslant \int_\Omega F(x,u)\, T_k(u-w) dx.$$

Theorem. *Suppose the following conditions are satisfied: (i) for a.e. $x \in \Omega$ the function $F(x,\cdot)$ is nonincreasing on $\mathbb{R}$; (ii) for any $s \in \mathbb{R}$ the function $F(\cdot,s)$ belongs to $L^1(\Omega)$. Then there exists an unique entropy solution of problem (1), (2).*

Proof is based on results represented in [1], [4].

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